

SYLLABUS FOR MATH 364 - ORDINARY DIFFERENTIAL EQUATIONS

BARD COLLEGE AT SIMON'S ROCK - SPRING 2014

Time: 2:25-3:20pm MWF

Email: clark@simons-rock.edu

Location: CL1-01

Office: 2nd Floor - Hall College Center

Instructor: Clark Musselman, Ph.D.

Office Phone: (413) 528-7292

1. COURSE WEBPAGE

The course website, located at <http://clarkmusselman.weebly.com/teaching.html>, will be updated with various documents pertaining to the course throughout the semester.

2. REQUIRED TEXT

Robinson, J., (2004). *An Introduction to Ordinary Differential Equations*. New York: Cambridge University Press. ISBN: 9780521533911.

3. OFFICE HOURS

Office hours are TBD. This time is reserved for you to talk to me about homework, exams, or anything else pertaining to the course. You do not need to make an appointment with me; you may simply stop by my office. If you have another class during my normally scheduled office hours, send me an email and we can make an appointment.

My office is on the second floor of Hall College Center, which is not wheelchair accessible. If a student needs to meet in an accessible area I will gladly make arrangements and find a suitable meeting place. If this is the case, please let me know via email.

4. COURSE DESCRIPTION

From the Simon's Rock Course Catalogue: *This is an introductory course on ordinary differential equations. Topics include first-order equations, second-order linear equations, harmonic oscillators, qualitative properties of solutions, power series methods, Laplace transforms, and existence and uniqueness theorems. Both the theory and applications are studied, including several problems of historical importance. Prerequisite: Mathematics 221 or permission of the instructor.*

5. EVALUATION

Your grade will be based on homework and class participation (20%), presentations (10%), three midterm exams (10% each), one team project (20%) and a final exam (20%). No grades will be dropped and there will be no extra credit. All work is due in class on the due date, however students may turn in work to me in person by 4:30pm without penalty *if I am available*. Students will not be reminded to turn work in during class. It is the students responsibility to remember to submit work to the instructor. Work delivered by campus mail will not be accepted. At the discretion of the instructor, late homework may be accepted for reduced credit.

There will be no make-ups given. If you must miss a quiz, exam, or final due to personal illness, a family emergency, or an official Simon's Rock sport or academic trip, you must provide official documentation (before the exam, or as soon as possible afterward if before-hand is impossible). If you do so, you may, at the discretion of the instructor, be excused from the missed quiz, exam, or final. Excuses will only be given with official documentation and only for the reasons listed above.

6. HOMEWORK

Completing homework assignments and checking your answers are the two most important things you can do to be successful in mathematics. Whether or not a given assignment is collected or graded, it is the responsibility of the student to complete *every* problem and to verify that *every* answer is correct. While it is expected that each student write solutions on their own, it is highly recommended that students work in groups, especially on difficult problems.

Eventually, every student will require some assistance in completing their homework. Help is available in many forms. The student can talk to their classmates, see a tutor, visit my office hours, or email me. **Do not let more than a day or two go by without getting your questions answered.**

Homework assignments are posted online and will **not** be announced in class. Regularly, some homework will be collected and given a 'check' or 'no-check'. The number of 'no-checks' a student receives will adversely affect the homework and class participation portion of their final grade.

When submitting homework, please make sure your pages are stapled together and that no additional assignments are included. If you prefer to write your homework in a notebook, you may turn in a photo-copy of the assignment, as long as it is your original work and in your own hand writing. No notebooks, email submissions, or typed work will be accepted without permission from the instructor.

Also, label each problem and leave plenty of space between them. This will allow your ideas to come across more clearly. If you find yourself crossing out much of your work, please re-write the page.

7. PRESENTATIONS

During the semester, each student will give two presentations, one short and one long. The short presentation will be a 5-8 minute explanation of a recent exam problem. Shortly after each exam, we will go over the exam in class. I will choose one student to present each exam problem on the board to the class and I will make every effort to give at least 48 hours notice. This will allow you to practice your presentation and to make corrections to your work, if need be. I am happy to meet with any student to discuss their work before they present. Each student will be chosen at least once during the semester to present an exam problem. If there are more students than exam problems, additional problems will be selected from the homework.

The longer and more involved presentation, will be done in groups of two and will last 30-50 minutes, depending on the topic and available class time. A list of available topics for the second presentation will be posted online later in the semester.

Your presentations will be graded not just on content, but also on clarity and completeness and will count toward your final grade. You will lose credit if you are not prepared or if you read from the text during your presentation. The rubric used to score your presentations is posted on the course website.

8. PROJECT

During the semester, one team project will be assigned. The content of the project will be posted online at a later date. Teams will be assigned by me and will include two to four students. You will be expected to work with your group members and *only* with members of your assigned group. The content of the project should not be discussed with members of other groups until after the project has been submitted to the instructor. **Discussing the project beforehand, with anyone other than your team mates or the instructor, will constitute academic dishonesty.** As always, I am more than happy to meet with individual students or with entire groups to discuss the projects. Time permitting, you will be given time in class to work with your groups.

Your final write-up must be in your own writing, legible, and be written in blue or black ink on the project packet and must show all necessary work. Typed solutions will not be accepted. Illegible solutions will be returned to the student ungraded. The student will then have the opportunity to rewrite and resubmit their work. Late submissions, including those returned for being illegible, will be given a 5% penalty per day, until I receive a legible and complete write-up.

9. EXAMS

Three midterm exams will be given throughout the semester. No calculators, notes, or books will be allowed on exams.

A final exam will be given at a date to be determined by the College. No calculators, notes, or books will be allowed. The final exam will be based on material from the entire course, although it will be weighted slightly more heavily on the material covered after the last midterm exam.

10. ATTENDANCE

Classes at Simon's Rock are interactive and all participants are adversely affected if one student is missing. As such, you are expected to attend every class. If you do miss a class, you are responsible for learning the material that was covered in your absence. You are also responsible for any quiz, exam, or in-class activity that you miss (see evaluation above). Initially, the student should contact a classmate to determine what material was missed. Only after consulting a classmate should the student contact the instructor for extra help on missed class material. Further, if you miss a week's worth of classes for any reason, The Office of Academic Affairs will be notified and you may be suspended from the course.

A student who arrives late or uses any electronic device (cell phone, laptop, etc.) during class will be considered *effectively* absent. Two effective absences will be treated as a single absence. Attendance will be taken once during class. If a student is not present when attendance is taken, it is the student's responsibility to check in with the instructor after class.

11. FURTHER INFORMATION

- Keep a copy of all your work before you turn it in so that nothing is lost in the unlikely event that papers go missing.

- You should expect to spend *at least* three to four hours working on this course outside of class for *each* hour in class.
- Academic honesty is valued at Simon's Rock. All students are expected to know and uphold the college's policies on academic dishonesty as described in the Catalogue.
- A student with special needs should feel welcome to discuss these with the instructor.
- Keep an updated copy of this syllabus. In the event that you transfer to another institution, this syllabus may be required for transfer credits to be accepted by your new institution.
- This syllabus is subject to change.
- Last updated 17:57 Monday 20th January, 2014.

Spring 2014 - Math 364 - ODE - Schedule

Day	Dates	Sections / Homework	Events
M	1/27	§1: 2, 3, 5	
W	1/29	§2: please read §3: 1. Also, plot (ix) in “dfield” and examine solns. §5: 1, 4, 5 (typo: $\dot{v} = -g$), 6, 7 (typo: “... is $V \cos \theta$...”)	§1 due
F	1/31	invited speaker	
M	2/3	§6: 1, 2, 3	§3 due
W	2/5	snow day - class canceled	
F	2/7	§6: WS1	§5 due
M	2/10	§6: WS2	
W	2/12	§7: 1-4, 6	WS1 due
F	2/14	§7 continued	
M	2/17	Exam 1	
W	2/19	Presentations	
F	2/21	§8: 1-4, 6 (typo: Ex 5.6), 10, 11 (Possible typo: Should be $\int_{x_0}^{\infty} \frac{1}{f(x)} dx$)	Last Day to Drop
M	2/24	§9: 1-3, 6, 8	
W	2/26	§10: 1i, 1iii, 3i, 4i, 5, 6	§8 due
F	2/28	§11: 1-3	
M	3/3	§4: 1i, 1iii, 1v, 2ii, 2iv	Class in Fisher
W	3/5	§21: 1, 2, 5, 8, Optional: 7	§9 due
F	3/7	WS3	Class in Fisher
M	3/10	Review	
W	3/12	Exam 2	
F	3/14	Presentations	
	3/15-3/30	Spring Break	
M	3/31	Project - Class in Fisher	
W	4/2	Project - Class in Fisher	Long Pres request due
F	4/4	§12: 1 (Do several, but not all. Check your solutions.), 2, 3 §19: 1 (Do several, but not all. Check your solutions.), 2	Handouts 1 & 2
M	4/7	§13: 1, 7, 8, 9, Optional: 5	VJR, EL, WM
W	4/9	§14: 1 (Do several, but not all. Check your solutions.), 2, 3	
F	4/11	§15: 1, 2, 3	Project due
M	4/14	§16: 1 (see page 151 for solving cubics)	
W	4/16	§17: 1, 2, 4, 5, 7, Optional: 8	
F	4/18	§18: §18: 1i-v (alternate formula and method), Optional: vi, vii	
M	4/21	§20: 1, 2i, 2ii, 2iv, 4, 5, 10	
W	4/23	§22: 1(i-iv), 4(i-iii)	JC, JB, WLL
F	4/25	§23: 1, 3, 6, 7	GG, IA, QC
M	4/28	Exam 3	
W	4/30	Presentations	Last Day to Withdraw
F	5/2	§24: 1, 2	MP, KT
M	5/5	§25: 1	
W	5/7	§26: 1 (as needed)	
F	5/9	§27: 1 (as needed)	
M	5/12	§28: 1, 2, 4 (For 2, just solve and draw.)	MP, DS, JY
W	5/14	§29: 1 (as needed), 2 (as needed), 4, 5i	NS, IG
T	5/20	Final Exam 12:00pm-2:00pm	

- “dfield and pplane”: <http://math.rice.edu/~dfield/dfpp.html>
- This schedule is subject to change. Check the course webpage frequently for updates.
- Last updated: 08:20 Thursday 17th April, 2014

Contents

<i>Preface</i>	<i>page</i>	<i>xiii</i>
Introduction		1
Part I First order differential equations		3
1 Radioactive decay and carbon dating		5
1.1 Radioactive decay		5
1.2 Radiocarbon dating		6
Exercises		8
2 Integration variables		9
3 Classification of differential equations		11
3.1 Ordinary and partial differential equations		11
3.2 The order of a differential equation		13
3.3 Linear and nonlinear		13
3.4 Different types of solution		14
Exercises		16
4 *Graphical representation of solutions using MATLAB		18
Exercises		21
5 ‘Trivial’ differential equations		22
5.1 The Fundamental Theorem of Calculus		22
5.2 General solutions and initial conditions		25
5.3 Velocity, acceleration and Newton’s second law of motion		29
5.4 An equation that we cannot solve explicitly		32
Exercises		33

Some of the chapters, and some sections within other chapters, are marked with an asterisk (*). These parts of the book contain material that either is more advanced, or expands on points raised elsewhere in the text.

6	Existence and uniqueness of solutions	38
6.1	The case for an abstract result	38
6.2	The existence and uniqueness theorem	40
6.3	Maximal interval of existence	41
6.4	The Clay Mathematics Institute's \$1 000 000 question	42
	Exercises	44
7	Scalar autonomous ODEs	46
7.1	The qualitative approach	46
7.2	Stability, instability and bifurcation	48
7.3	Analytic conditions for stability and instability	49
7.4	Structural stability and bifurcations	50
7.5	Some examples	50
7.6	The pitchfork bifurcation	54
7.7	Dynamical systems	56
	Exercises	56
8	Separable equations	59
8.1	The solution 'recipe'	59
8.2	The linear equation $\dot{x} = \lambda x$	61
8.3	Malthus' population model	62
8.4	Justifying the method	64
8.5	A more realistic population model	66
8.6	Further examples	68
	Exercises	72
9	First order linear equations and the integrating factor	75
9.1	Constant coefficients	75
9.2	Integrating factors	76
9.3	Examples	78
9.4	Newton's law of cooling	79
	Exercises	86
10	Two 'tricks' for nonlinear equations	89
10.1	Exact equations	89
10.2	Substitution methods	94
	Exercises	97
Part II	Second order linear equations with constant coefficients	99
11	Second order linear equations: general theory	101
11.1	Existence and uniqueness	101
11.2	Linearity	102
11.3	Linearly independent solutions	104
11.4	*The Wronskian	106

11.5	*Linear algebra	107
	Exercises	109
12	Homogeneous second order linear equations	111
12.1	Two distinct real roots	112
12.2	A repeated real root	113
12.3	No real roots	115
	Exercises	118
13	Oscillations	120
13.1	The spring	120
13.2	The simple pendulum	122
13.3	Damped oscillations	123
	Exercises	126
14	Inhomogeneous second order linear equations	131
14.1	Complementary function and particular integral	131
14.2	When $f(t)$ is a polynomial	133
14.3	When $f(t)$ is an exponential	135
14.4	When $f(t)$ is a sine or cosine	137
14.5	Rule of thumb	139
14.6	More complicated functions $f(t)$	139
	Exercises	140
15	Resonance	141
15.1	Periodic forcing	141
15.2	Pseudo resonance in physical systems	145
	Exercises	148
16	Higher order linear equations	150
16.1	Complementary function and particular integral	150
16.2	*The general theory for n th order equations	152
	Exercises	153
Part III	Linear second order equations with variable coefficients	157
17	Reduction of order	159
	Exercises	162
18	*The variation of constants formula	164
	Exercises	168
19	*Cauchy–Euler equations	170
19.1	Two real roots	171
19.2	A repeated root	171
19.3	Complex roots	173
	Exercises	174

20	*Series solutions of second order linear equations	176
20.1	Power series	176
20.2	Ordinary points	178
20.3	Regular singular points	183
20.4	Bessel's equation	187
	Exercises	195
Part IV	Numerical methods and difference equations	199
21	Euler's method	201
21.1	Euler's method	201
21.2	An example	203
21.3	*MATLAB implementation of Euler's method	204
21.4	Convergence of Euler's method	206
	Exercises	209
22	Difference equations	213
22.1	First order difference equations	213
22.2	Second order difference equations	215
22.3	The homogeneous equation	215
22.4	Particular solutions	219
	Exercises	222
23	Nonlinear first order difference equations	224
23.1	Fixed points and stability	224
23.2	Cobweb diagrams	225
23.3	Periodic orbits	226
23.4	Euler's method for autonomous equations	227
	Exercises	230
24	The logistic map	233
24.1	Fixed points and their stability	234
24.2	Periodic orbits	234
24.3	The period-doubling cascade	237
24.4	The bifurcation diagram and more periodic orbits	238
24.5	Chaos	240
24.6	*Analysis of $x_{n+1} = 4x_n(1 - x_n)$	242
	Exercises	245
Part V	Coupled linear equations	247
25	*Vector first order equations and higher order equations	249
25.1	Existence and uniqueness for second order equations	251
	Exercises	252
26	Explicit solutions of coupled linear systems	253
	Exercises	257

27	Eigenvalues and eigenvectors	259
27.1	Rewriting the equation in matrix form	259
27.2	Eigenvalues and eigenvectors	260
27.3	*Eigenvalues and eigenvectors with MATLAB	266
	Exercises	267
28	Distinct real eigenvalues	269
28.1	The explicit solution	270
28.2	Changing coordinates	271
28.3	Phase diagrams for uncoupled equations	276
28.4	Phase diagrams for coupled equations	279
28.5	Stable and unstable manifolds	281
	Exercises	282
29	Complex eigenvalues	285
29.1	The explicit solution	285
29.2	Changing coordinates and the phase portrait	287
29.3	The phase portrait for the original equation	291
	Exercises	292
30	A repeated real eigenvalue	295
30.1	\mathbb{A} is a multiple of the identity: stars	295
30.2	\mathbb{A} is not a multiple of the identity: improper nodes	295
	Exercises	299
31	Summary of phase portraits for linear equations	301
31.1	*Jordan canonical form	301
	Exercises	305
Part VI	Coupled nonlinear equations	307
32	Coupled nonlinear equations	309
32.1	Some comments on phase portraits	309
32.2	Competition of species	310
32.3	Direction fields	311
32.4	Analytical method for phase portraits	314
	Exercises	322
33	Ecological models	323
33.1	Competing species	323
33.2	Predator-prey models I	331
33.3	Predator-prey models II	334
	Exercises	338
34	Newtonian dynamics	341
34.1	One-dimensional conservative systems	341
34.2	*A bead on a wire	344

34.3	Dissipative systems	347
	Exercises	350
35	The ‘real’ pendulum	352
35.1	The undamped pendulum	352
35.2	The damped pendulum	356
35.3	Alternative phase space	358
	Exercises	358
36	*Periodic orbits	360
36.1	Dulac’s criterion	360
36.2	The Poincaré–Bendixson Theorem	361
	Exercises	362
37	*The Lorenz equations	364
38	What next?	373
38.1	Partial differential equations and boundary value problems	373
38.2	Dynamical systems and chaos	374
	Exercises	375
	<i>Appendix A Real and complex numbers</i>	379
	<i>Appendix B Matrices, eigenvalues, and eigenvectors</i>	382
	<i>Appendix C Derivatives and partial derivatives</i>	387
	<i>Index</i>	395